Gustav Schmidt-Mitt.

ELEMENTS OF MECHANISM.

Pantograph Extract

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In Fig. 177, letting the angle $ada_1 = \theta$ and $bcb_1 = \phi$, we have, from equation (50),

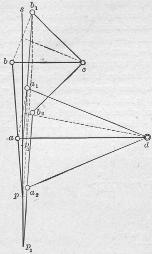

FIG. 178.

$$\frac{ap}{bp} = \frac{ag}{bh} = \frac{ae}{bf}$$

$$= \frac{ad(1-\cos\theta)}{bc(1-\cos\phi)} = \frac{ad\ 2\sin^2\frac{\theta}{2}}{bc\ 2\sin^2\frac{\phi}{2}},$$

which may be written

$$\frac{ap}{bp} = \frac{bc}{ad} \times \frac{\overline{ad}^2 \sin^2\frac{\theta}{2}}{\overline{bc}^2 \sin^2\frac{\phi}{2}}.$$

But $ad \sin\theta = bc \sin\phi$; and since the angles θ or ϕ would rarely exceed 20°, we may assume that $ad \sin\frac{\theta}{2} = bc \sin\frac{\phi}{2}$.

$$\therefore\ \frac{ap}{bp} = \frac{bc}{ad}, \text{ nearly}, \quad . \quad . \quad . \quad (51)$$

or the segments of the link are inversely proportional to the lengths of the nearer levers, which is the rule usually employed when the extreme positions can vary a very little from the straight line. When the levers are equal this rule is exact.

129. The Pantograph.—The pantograph is a four-bar linkage so arranged as to form a parallelogram *abcd*, Fig. 179. Fixing some point in the linkage, as *e*, certain other points, as *f*, *g*, and *h*, will move parallel and similar to each other over any path either straight or curved. *These points, as f, g, and h, must lie on the same straight line passing through the fixed point e, and their motions will then be proportional to their distances from the fixed point.* To prove that this is so, move the point *f* to any other position, as f_1; the linkage will then be found to occupy the position $a_1b_1c_1d_1$. Connect f_1 with *e*;

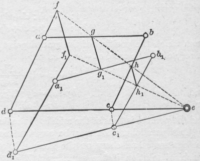

FIG. 179.

then h_1, where f_1e crosses the link b_1c_1, can be proved to be the same distance from c_1 that h is from c, and the line hh_1 will be parallel to ff_1.

In the original position, since fd is parallel to hc, we may write

$$\frac{fd}{hc}=\frac{de}{ce}=\frac{fe}{he}.$$

In the second position, since f_1d_1 is parallel to h_1c_1 and since f_1e is drawn a straight line, we have

$$\frac{f_1d_1}{h_1c_1}=\frac{d_1e}{c_1e}=\frac{f_1e}{h_1e}.$$

Now in these equations $\frac{de}{ce}=\frac{d_1e}{c_1e}$; therefore $\frac{fd}{hc}=\frac{f_1d_1}{h_1c_1}$; but $fd=f_1d_1$, which gives $hc=h_1c_1$, which proves that the point h has moved to h_1. Also $\frac{fe}{he}=\frac{f_1e}{h_1e}$, from which it follows that ff_1 is parallel to hh_1, and

$$\frac{ff_1}{hh_1}=\frac{fe}{he}=\frac{de}{ce},$$

or the motions are proportional to the distances of the points f and h from e.

To connect two points, as a and b, Fig. 180, by a pantograph, so that their motions shall be parallel and similar and in a given ratio, we have, first, that the fixed point c must be on the straight line ab continued, and so located that ac is to bc as the desired ratio of the motion of a to b. After locating c, an infinite number of pantographs might be drawn. Care must be taken that the links are so proportioned as to allow the desired magnitude and direction of motion.

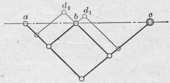

FIG. 180.

It is interesting to note that if b were the fixed point, a and c would move in opposite directions. It can be shown as before that their motions would be parallel and as ab is to bc.

The pantograph is often used to reduce or enlarge drawings, for it is evident that similar curves may be traced as well as straight lines. Also pantographs are used to increase or reduce motion in some definite proportion, as in the indicator rig on an engine where the motion of the cross-head is reduced proportionally to the desired length of the indicator diagram. When the points, as f and h (Fig. 179), are required to move in parallel straight lines it is not always necessary to employ a complete parallelogram, provided the mechanism is such that the points *f* and *h* are properly guided. Such a case is shown in Fig. 181, which is a diagram of the mechanism for moving the pencil on a Thompson steam-engine indicator. The pencil at *f*, which traces the diagram on a paper carried by an oscillating drum, is guided by a Scott-Russell straight-line motion *abcd* so that it moves nearly in a straight line *ss* parallel to the axis of the drum, and to the centre line of the cylinder *tt*. It must also be arranged that the motion of the pencil *f* always bears the same relation to the motion of the piston of the indicator on the line *tt*. To secure this draw a line from *f* to *d* and note the point *e* where it crosses the line *tt*: *e* will be a point on the piston-rod, which

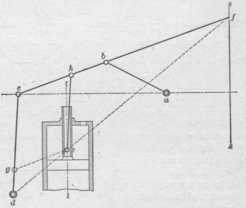

FIG. 181.

rod is guided in an exact straight line by the cylinder. If now the link *eh* is added so that its centre line is parallel to *cd*, we should have, assuming *f* to move on an exact straight line, the motion of *f* parallel to the motion of *e* and in a constant ratio as $cf:ch$ or as $df:de$. This can be seen by supposing the link *eg* to be added, which completes the pantograph *dgehcf*. If *eg* were added, the link *ab* could not be used, as the linkage *abcdf* does not give an exact straight-line motion to *f*. For constructive reasons the link *eg* is omitted; a ball joint is located at *e* which moves in an exact straight line, and the point *f* is guided by the Scott-Russell motion, the error in the motion being very slight indeed.

Slides are often substituted, in the manner just explained, for links of a pantograph, and exact reductions are thereby obtained. In Fig. 182 the points *f* and *h* are made to move on the parallel lines *mm* and *nn* respectively. Suppose it is desired to have the point *h* move $\frac{1}{3}$ as much as *f*. Draw the line *fhe* and lay off the point *e* so that $eh:ef=1:3$; draw a line, as ed, and locate a point d upon it which when connected to f with a link df will move nearly an equal distance to the right and left of the line ef and above and below the line mm for the known motion of f. Draw ch through h and parallel to df. The linkage $echdf$ will accomplish the result required. The dotted link ah may be added to complete the pantograph, and the slide h may then be removed or not as desired. The figure also shows how a point g may be made to move in the opposite direction to f in the same ratio as h but on the line n_1n_1, the equivalent pantograph being drawn dotted. The link ed is shown in its extreme position to the left by heavy lines and to the right by light lines.

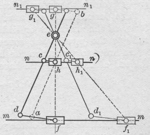

FIG. 182.

130. Applications of Watt's Parallel Motion. — Watt's parallel motion has been much used in beam engines, and it is generally necessary to arrange so that more than one point can be guided, which is accomplished by a pantograph attachment.

In Fig. 183 a parallel motion is arranged to guide three points p, p_1, and p_2 in parallel straight lines. The case chosen is that of a compound condensing beam engine, where P_2 is the piston-rod of the low-pressure cylinder, P_1 that of the high-pressure cylinder, and P the pump-rod, all of which should move in parallel straight lines, perpendicular to the centre line of the beam in its middle position.

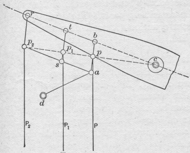

FIG. 183.

The fundamental linkage $dabc$ is arranged to guide the point p as required; then adding the parallelograms $astb$ and ap_2rb, placing the links st and p_2r so that they pass through the points p_1 and p_2 respectively, found by drawing the straight line cp and noting points p_1 and p_2 where it intersects lines P_1 and P_2, we obtain the complete linkage. The links are arranged in two sets, and the rods are carried between them; the links da are also placed outside of the links p_2a. When the point p falls within the beam a double pump-rod must be used. The linkage is shown in its extreme upper position to render its construction clearer.

The various links are usually designated as follows: cr the *main beam*, ad the *radius-bar* or *bridle*, p_2r the *main link*, ab the *back link*, and p_2a the *parallel bar*, connecting the main and back links.

In order to proportion the linkage so that the point p_2 shall fall at the end of the link rp_2 we have, by similar triangles cbp and crp_2,

$$cb : bp = cr : rp_2 = cr : ab.$$

$$\therefore \quad cr = \frac{ab \times cb}{bp}.$$

The relative stroke S of the point p_2 and s of the point p are expressed by the equation

$$S:s = cp_2:cp = cr:cb.$$

If we denote by M and N the lengths of the perpendiculars dropped from c to the lines of motion P_2 and P respectively, then

$$S:s = M:N$$

and

$$S = s\frac{M}{N}; \quad s = S\frac{N}{M}. \quad . \ . \ . \ . \ . \ . \ . \quad \text{(A)}$$

The problem will generally be, given the centres of the main beam c and bridle d, the stroke S of the point p_2, and the paths of the guided points p, p_1, and p_2, to find the remaining parts. The strokes of the guided points can be found from equation (A) and then the method of § 128, Fig. 177, can be applied.

131. Roberts's Approximate Straight-line Motion.—This might also be called the W straight-line motion, and is shown in Fig. 184. It consists of a rigid triangular frame abp forming an isosceles triangle on ab, the points a and b being guided by links $ad = bc = bp$, oscillating on the centres d and c respectively, which are on the line of motion dc.

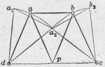

FIG. 184.

To lay out the motion, let dc be the straight line of the stroke along which the guided point p is to move approximately, and p be the middle point of that line. Draw two equal isosceles triangles, dap and cbp; join ab, which must equal $dp = pc$. Then abp is the rigid triangular frame, p the guided point, and d and c are the centres of the two links. The extreme positions when p is at d and c are shown at da_1a_2 and ca_2b_2, the point a_2 being common to both. The length of each side of the triangle, as $ap = da$, should not be less than 1.186 dp, since in this case the points ca_2a_1 and da_2b_2 lie in straight lines.